Comparison Study of Finite Element Methods Dealing with Floating Conductors in Electric Field

W. N. Fu[1], S. L. Ho[1], Shuangxia Niu[1], and Jianguo Zhu[2]
[1]Department of Electrical Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong
[2]Faculty of Engineering, University of Technology, Sydney, P.O. Box 123, Broadway NSW 2007, Australia
eewnfu@polyu.edu.hk

Abstract — In transient magnetic field computation, it is highly desirable if the stray capacitances among all conductors can be computed conveniently and their effects can be addressed using magnetic field – circuit coupling. Because of the existence of floating conductors, the general finite element (FE) solver of Poisson's equation, which is used by magnetic field solver, cannot be used to extract the capacitance matrix directly. In this paper, methods to deal with floating conductors in electrostatic field for field solution and capacitance matrix extraction using finite element method (FEM) are compared; their merits and shortcomings are discussed. A FE formulation to deal with charge-excitations on conductors is deduced. A method to compute the electric field and extract capacitance matrix to include the effect of floating conductors inside the solution domain is put forward, and the merit of the proposed algorithm is that the general FE solver can be used without the need for any program modification.

I. INTRODUCTION

To simulate the power electronic electric drives, it is important to address high-frequency effects in the solution because of the high speed switching and repetitive steep rising and falling in the voltage waveforms [1-3]. In addition, solving full-wave electromagnetic field that includes stray capacitances is very time consuming. A practical and promising method is to couple the stray capacitances among conductors with the transient magnetic field using field – circuit coupled method [4]. It is highly desirable if the general finite element (FE) solver of Poisson's equation, which is used by the magnetic field solver, can also be used to extract the capacitance matrix. However, because of the existence of floating conductors which represent equipotential volumes but which do not constitute the Dirichlet boundary conditions, a special numerical constraint needs to be implemented. Reference [5] presents the constraint equations for floating conductors in finite-difference methods for determining the capacitance relationships among conductors. The method used in [6-8] is to regard floating conductors as dielectric materials having infinite permittivity. In actual program implementation, infinity is realized using a very large number, and numerical errors from the aforementioned formulation become inevitable.

In this paper a simple and direct method to deal with floating conductors inside the solution domain for the electric field finite element method (FEM) using electric charge excitations is presented first. The equivalent capacitance coefficient matrix including the effect of floating conductors can then be extracted. The merits of such approach are that the floating conductors are automatically taken as equipotential objects even though they are in the solution domain; the effect of floating conductors, which are not the circuit ports, can be taken into account in the extracted lumped capacitances. Another advantage is that for different charge excitations on the conductors, only the right-hand side (RHS) of the system equation changes, whereas the coefficient matrix remains unchanged. Hence a multi-RHS solver can be used to reduce the computing time.

The shortcoming of the charge-excitation method is that it requires complicated internal modifications to existing FE programs. The voltage-excitation method needs further attention. The proposed method is to compute the complete capacitance matrix first. For the computation of the capacitances among the floating conductors and other conductors, each floating conductor will have its own voltage excitation or zero-value Dirichlet boundary condition. The complete capacitance matrix can be obtained without dealing with the floating conductors. Then the entries of the floating conductors are removed and a reduced-size matrix is automatically obtained. Lastly, from the charge-potential relationship, the potentials on the floating conductors can be obtained and the field distribution of the nominal problem can be computed using the voltage excitation method again. This method does not require any modification of normal FE programs, and the multi-RHS solver can be used to reduce the computing time.

II. CHARGE-EXCITATION METHOD

A. Computation of Field

For a two-dimensional (2D) problem on x-y plane, discritizing the field equation using Galerkin method gives

$$\iint_{\Omega} \varepsilon(\nabla W \cdot \nabla V)\mathrm{d}x\mathrm{d}y = \iint_{\Omega} W\sigma \mathrm{d}x\mathrm{d}y + \int_l \varepsilon W \frac{\partial V}{\partial n}\mathrm{d}c, \tag{1}$$

where, W is a weighting function, edge l is the boundary.

For a conductor with a total electric charge Q, the conductor region is denoted as Ω^+ and is bounded by edge l. The electric charge is always on the side l^+ of the conductor. The last term in (1) is:

$$\int_l \varepsilon W \frac{\partial V}{\partial n}\mathrm{d}c = \int_{l^+}\left(\varepsilon W \frac{\partial V}{\partial n}\right)^+ \mathrm{d}c - \int_{l^-}\left(\varepsilon W \frac{\partial V}{\partial n}\right)^- \mathrm{d}c = \int_{l^+} W\rho_S \mathrm{d}c, \tag{2}$$

where ρ_S is the surface charge density. The line integration of (2) will only be on the edge of the conductor.

B. Computation of Capacitance Matrix

Here the charge-excitation method is presented to compute the capacitance matrix. Because of the relationship of $\boldsymbol{Q} = \boldsymbol{CV}$, V_1, V_2, …, V_N representing the potentials of the N conductors:

$$\begin{Bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{Bmatrix} = \begin{bmatrix} C_{11} & -C_{12} & \cdots & -C_{1N} \\ -C_{21} & C_{22} & \cdots & -C_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ -C_{N1} & -C_{N2} & \cdots & C_{NN} \end{bmatrix}^{-1} \begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_N \end{Bmatrix} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_N \end{Bmatrix}. \quad (3)$$

G matrix is the elastance matrix, If the charge excitations are set as, $\{Q_1 \quad Q_2 \quad \cdots \quad Q_N\}^T = \{1 \quad 0 \quad \cdots \quad 0\}^T$, one has $\{G_{11} \quad G_{21} \quad \cdots \quad G_{N1}\}^T = \{V_1 \quad V_2 \quad \cdots \quad V_N\}^T$. By using a similar method, G_{1N}, G_{2N}, G_{2N}, …, and G_{NN} can all be computed.

After the G matrix is obtained, the C matrix can be obtained. With this algorithm, the coefficient matrix of the FE equations can be kept unchanged. Only a multi right hand side (RHS) problem needs to be solved. By using the multi-RHS algebraic solvers, the computing time required to extract the capacitance matrix can be greatly reduced.

III. VOLTAGE-EXCITATION METHOD

A. Computation of Complete Capacitance Matrix

Consider an N-conductor system, the electrostatic stored energy can be computed by

$$W = \frac{1}{2}\{V_1 \quad V_2 \quad \cdots \quad V_N\} \begin{bmatrix} C_{11} & -C_{12} & \cdots & -C_{1N} \\ -C_{21} & C_{22} & \cdots & -C_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ -C_{N1} & -C_{N2} & \cdots & C_{NN} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{Bmatrix}. \quad (4)$$

For the computation of self capacitances, if the voltage excitations are set as: $\{V_1 \quad V_2 \quad \cdots \quad V_N\}^T = \{1 \quad 0 \quad \cdots \quad 0\}^T$, one has $C_{11} = 2W_1$. For the computation of mutual capacitances, if the voltage excitations are set as, $\{V_1 \quad V_2 \quad V_3 \quad \cdots \quad V_N\}^T = \{1 \quad 1 \quad 0 \quad \cdots \quad 0\}^T$, one has, $C_{12} = (-2W_{12} + C_{11} + C_{22})/2$. By using a similar method, C_{13}, C_{21}, C_{23}, …, and $C_{(N-1)N}$ can all be computed.

Each Dirichlet boundary condition is assembled to the RHS of the system equation. Therefore, the principle of linear superposition can also be applied. If the system has N conductors, the capacitances are computed by solving the system matrix equation N times with the same coefficient matrix. Multi-RHS solvers can also be used to reduce the computing time.

B. Matrix Reduction Method for Floating Conductors

For a floating conductor, the total charge on the conductor is zero. The row and column corresponding to that conductor in the capacitance matrix can be eliminated without solving FE equations.

C. Computation of Nominal Field

If the original problem has conductor charge excitations or has floating conductors, according to the Q-V relationship, the potentials on the conductors can be obtained. Then the voltage excitations are applied to all conductors and the field solution of the nominal problem can be computed.

IV. TEST CASE

A 4-conductor system above a ground plane, where an analytical solution is available, is used as a test case [9].

The capacitance matrix computed by using analytical method is:

$$C = \begin{bmatrix} 14.840556 & -3.8479870 & -0.93025803 & -0.44076465 \\ -3.8479870 & 15.825204 & -3.6344104 & -0.93025803 \\ -0.93025803 & -3.6344104 & 15.825204 & -3.8479870 \\ -0.44076465 & -0.93025803 & -3.8479870 & 14.840556 \end{bmatrix} \text{(pF)}. \quad (5)$$

The capacitance matrix computed by using the voltage-excitation FEM is:

$$C = \begin{bmatrix} 14.851 & -3.8486 & -0.93902 & -0.44385 \\ -3.8486 & 15.842 & -3.6324 & -0.93902 \\ -0.93902 & -3.6324 & 15.842 & -3.8487 \\ -0.44385 & -0.93902 & -3.8487 & 14.851 \end{bmatrix} \text{(pF)}. \quad (6)$$

If conductors 2 and 3 are floated, the capacitance matrix computed by applying the matrix reduction method to (6) is:

$$C = \begin{bmatrix} 13.69499 & -1.16517 \\ -1.16517 & 13.69493 \end{bmatrix} \text{(pF)}. \quad (7)$$

The capacitance matrix computed by using the charge-excitation method of the floating conductors is:

$$C = \begin{bmatrix} 13.695 & -1.1652 \\ -1.1652 & 13.695 \end{bmatrix} \text{(pF)}. \quad (8)$$

Comparing the results in (5) and (6), also in (7) and (8), it can be seen that both proposed methods are giving precise solutions.

V. REFERENCES

[1] L. Gubbala, A. von Jouanne, P.N. Enjeti, C. Singh, H. A. Toliyat, "Voltage distribution in the windings of an ac motor subjected to high dv/dt PWM voltages," IEEE PESC Conf. Proc., pp. 579-585, 1995.

[2] D. B. Hyypio, "Simulation of cable and winding response to steep-fronted voltage waves," IEEE IAS Conf. Rec., pp. 800-806, 1995.

[3] Y. Tang, "Analysis of steep-fronted voltage distribution and turn insulation failure in inverter fed ac motor," IEEE IAS Conf. Rec., pp. 509-516, 1997.

[4] W. N. Fu, P. Zhou, D. Lin, S. Stanton and Z. J. Cendes, "Modeling of solid conductors in two-dimensional transient finite-element analysis and its application to electric machines," IEEE Trans. Magn., vol. 40, no. 2, pp. 426-434, 2004.

[5] C. N. Dorny and R. A. Mathias, "Finite-difference methods for determining capacitance relationships among arbitrarily shaped conductors," IEEE Trans. Power Apparatus and Systems, vol. PAS-90, no. 2, pp. 876 – 881, 1971.

[6] A. Konrad and M. Graovac, "The floating potential approach to the characterization of capacitive effects in high-speed interconnects," IEEE Trans. Magn., vol. 33, no. 2, pp. 1185 - 1188, 1997.

[7] A. Konrad and M. Graovac, "The finite element modeling of conductors and floating potentials," IEEE Trans. Magn., vol. 32, no. 5, pp. 4329 - 4331, 1996.

[8] G. J. Cokkinides, B. Beker and A. Templeton, "Direct computation of capacitance in integrated passive components containing floating conductors," IEEE Trans. Components, Packaging, and Manufacturing Technology, part B, vol. 20, no. 2, pp. 123 - 128, 1997.

[9] P. Silvester, Modern Electromagnetzc Fzelds, Englewood Cliffs, N.J.: Prentice-Hall, 1968, Ch.2.